Ay 10 - Problem Set #11
LAST ONE FOR THE SEMESTER!!!!
Due: December 6, 11am

Please write your full name, section number, and GSI's name at the top of your homework. Also, be sure to put your homework in the correct box in the basement of Campbell Hall. There is a 20% penalty per day for the late submission of assignments, however you do get one "freebie" (see course syllabus for more info).

Remember to always show your work; no credit will be given for just a final answer. However, if you get most of the question right but get the final answer wrong, you will get most of the points. Use a calculator where necessary and dont forget units if the answer requires them.

If you use any resource besides the textbook, lecture, or section (*e.g.* a web site), be sure to include proper attribution for the reference. Feel free to work with other students in the class, but remember that all work turned in must be your own (*i.e.* don't just copy the work of another student).

1. **(5 points) Supernovae are Super!**

   Typical type Ia supernovae have a luminosity of about $10^8 L_\odot$. In comparison, the luminosity of a bright Cepheid variable is about $10^4 L_\odot$. The Hubble Space Telescope is sensitive enough to observe Cepheid variables out to a distance of about 100 million light-years.

   (a) Estimate the furthest distance to a typical type Ia supernova that Hubble can observe.

   (b) A decent approximation for the age of the universe is 13.6 billion years. Using this fact, estimate the size of the observable universe in light-years. (NOTE: This should be **very** straightforward.)

   (c) How does the distance you calculated in part (a) compare to the size of the observable universe you calculated in part (b)?

2. **(6 points) Questions on Quasars.**

   The luminosity of a quasar is typically about $10^{12} L_\odot$, which is about ten times larger than the luminosity of the host galaxy. Astronomers believe that this energy is released by material falling in to the supermassive black hole at the center of the galaxy.

   Analyzing the energy released by matter falling into a black hole can get rather complicated. In this problem we'll simplify things and assume that when a mass M falls into the central black hole, the enery released is about $E = 0.1Mc^2$ (this should look familiar, it's just Einstein's famous equation with an efficiency of 10%).

   (a) How many solar masses per year must the supermassive black hole in a quasar consume to generate the observed luminosity?

(b) Suppose the host galaxy initially had a mass of $10^{11} M_\odot$. How long would it take for the entire galaxy to be consumed by the central black hole?

NOTE: Astronomers do not think that these supermassive black holes will actually consume their entire host galaxy.

3. **(12 points) Edwin Hubble, I'm your density. I mean...your destiny.**

Recall that the **critical density** is the density corresponding to a universe in which *gravity acting alone* would be just sufficient to halt the present expansion.

(a) The value of the critical density is about $9 \times 10^{-27} \frac{\text{kg}}{\text{m}^3}$. What is it in solar masses per cubic parsec?

(b) Picture a huge sphere centered around the Sun. If we keep making the sphere larger and larger (still keeping the Sun at its center), then the sphere will eventually run into the nearest star. When this happens, what is the radius of the sphere?

(c) Using your radius from above, what is the approximate density of matter inside the sphere?

(d) Compare your answers to part (a) and the previous question. Do we live in a special (i.e. bizarrely over or under-dense) part of the universe?

For the rest of this problem, assume that the critical density is equal to the density of just "normal" matter (as opposed to "normal" matter *plus* dark matter), namely, about $10^{-28} \frac{\text{kg}}{\text{m}^3}$.

(e) If the crtical density is proportional to the square of Hubble's constant, what would the new value of Hubble's constant be (using a critical density equal to the density of normal matter)?

(f) Is the Hubble constant calculated above within the currently accepted range of values?

4. **(16 points) It's the End of the Universe As We Know It.**

For the purposes of this problem, assume that there is no cosmological constant and that the only thing controlling the evolution of the scale factor of the universe with cosmic time is expansion and deceleration due to gravity.

(a) Sketch the age of the universe (on the horizontal axis) versus the scale factor of the universe (on the vertical axis). Make sure that all the universes were "born" (*i.e.* have the scale factor equal to zero) at the same time. Label your axes (scale factor, R, and age, t), but you do not need to put numbers on the axes.

i. Universe A, which has $\Omega_m = 0.1$.

ii. Universe B, which has $\Omega_m = 1$.

iii. Universe C, which has $\Omega_m = 100$.

iv. Universe D, which has $\Omega_m = 1000$.

(b) Which universe(s) "live" (*i.e.* have the scale factor greater than 0) for the least amount of time?

(c) Which universe(s) live forever?

(d) Which universe(s) will have a "Big Crunch"?

(e) At a given time after the Big Bang, which universe(s) are the largest?

(f) In which universe(s) will you, if you live long enough, eventually measure a cosmological **blue**shift?

(g) For the universe(s) you listed for the previous question, label the point on their "age versus scale factor" graphs where people would STOP measuring cosmological **red**shifts and START measuring cosmological **blue**shifts.

IT'S OVER!!!