Ay 10 - Problem Set #8
Due: November 1, 11am

Please write your full name, section number, and GSI's name at the top of your homework. Also, be sure to put your homework in the correct box in the basement of Campbell Hall. There is a 20% penalty per day for the late submission of assignments, however you do get one "freebie" (see course syllabus for more info).

Remember to always show your work; no credit will be given for just a final answer. However, if you get most of the question right but get the final answer wrong, you will get most of the points. Use a calculator where necessary and don't forget units if the answer requires them.

If you use any resource besides the textbook, lecture, or section (*e.g.* a web site), be sure to include proper attribution for the reference. Feel free to work with other students in the class, but remember that all work turned in must be your own (*i.e.* don't just copy the work of another student).

1. **(6 points + 2 extra credit)** An Exploding Crab!

   Figure 1 is a picture of the Crab Nebula, the remnant of a supernova. The white line in the upper left-hand corner of Figure 1 shows the scale of the figure. Observations of the Crab Nebula taken over several decades show that the *circled* gas blob is moving away from the center of the nebula (marked by the black X) at about 2.60×10^{-3} arcminutes per year.

   (a) Assuming the gas blob has been traveling at a constant velocity the whole time, use the information above to estimate the year in which the supernova explosion would have been seen on the Earth.

   (b) How does your answer compare with the year in which the supernova that produced this nebula was first observed — is it larger, smaller, or the same? If it's not the same, offer a physics-based explanation for why your answer differs the way that it does.

   (c) The Crab Nebula is about 6300 lightyears away from the Earth. What is the actual distance, measured in lightyears, between the circled blob of gas and the marked center of the nebula? Recall that if θ is a small angle *measured in radians*, $\tan\theta \approx \theta$. **Extra credit:** Compare the speed of a supernova remnant, exemplified by the circled blob of gas, to the speed of an expanding planetary nebula shell. A typical value for the latter quantity can be found in the textbook.

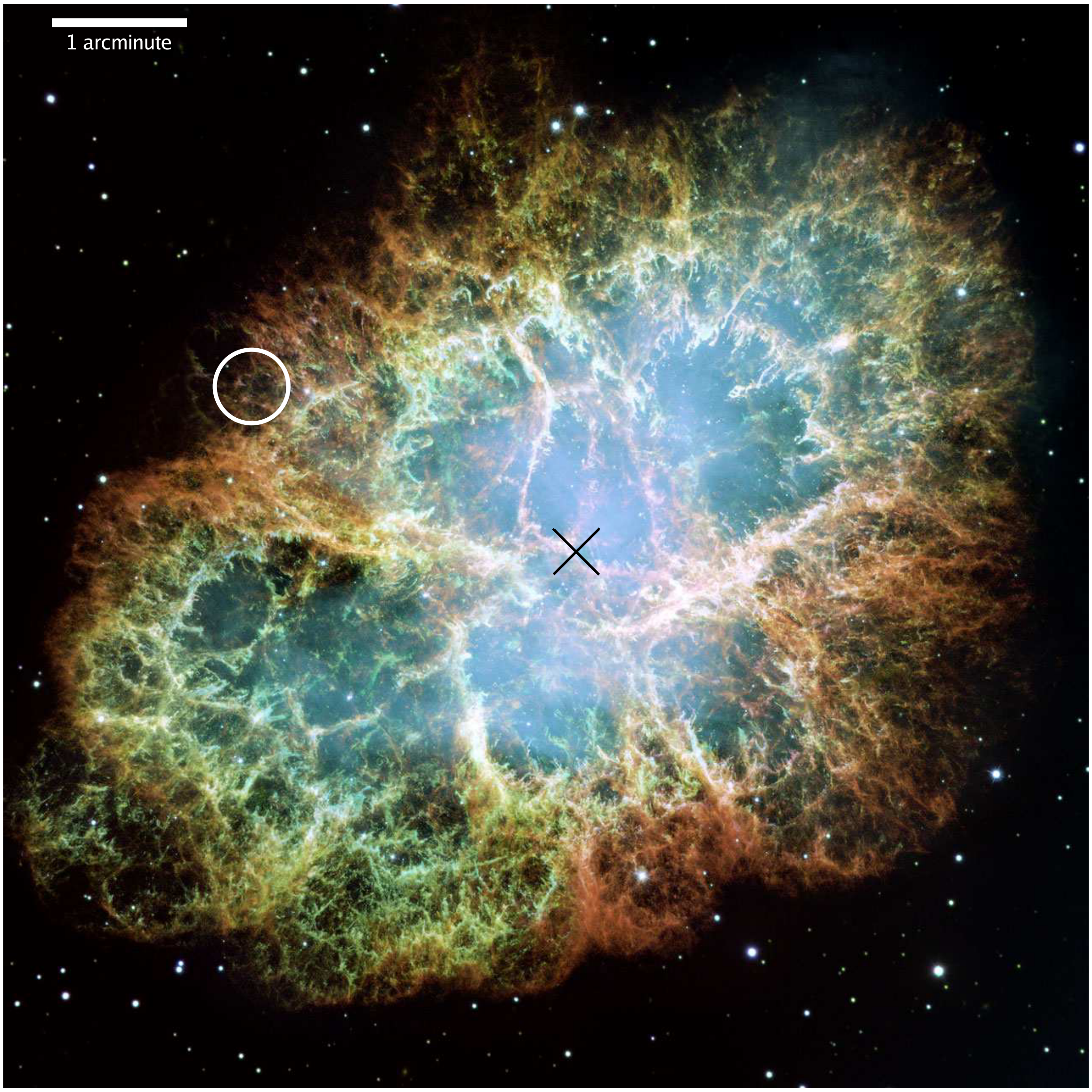

Figure 1: The Crab Nebula. The black X represents the center of the nebula and the white circle marks a moving blob of gas.

2. **(6 points)** Powerful, Powerfuller, Powerfullest

   (a) The Sun's life on the main sequence will last about 10^{10} years. What is the total amount of energy that the Sun will release over its main sequence lifetime? Assume that while on the main sequence, the Sun always shines at its current luminosity.

   (b) A Type II supernova happens when a large, massive star collapses and converts its gravitational energy into an explosion. The total energy released in such a supernova is thus about equal to the gravitational energy of the star, which is given by the equation $E_{\text{grav}} = GM^2/R$, where M is the star's mass, R its radius, and G the gravitational constant. How much energy would be released if the blue supergiant Rigel were to explode? Read page 531 of the textbook to find the numbers you need. How does your answer compare to the total energy that the Sun will release over its whole life?

   (c) A gamma ray burst (GRB) is another kind of powerful cosmic explosion. Typically a GRB emits 10^{45} J of energy in just a few seconds. (Recall that a supernova emits its energy over a few months.) How does this number compare to your answers to the first two parts?

3. **(5 points)** Mmmmm, Toasty.

   A supernova occurring very near to the Earth could make life quite uncomfortable here.

   (a) At its brightest, a Type I supernova has a luminosity of about 4×10^9 $L_\odot$. How far would a Type I supernova need to be from the Earth in order to appear ten times brighter than the Sun? Use the inverse-square law for brightness, $B = L/4\pi d^2$. Give your answer in parsecs.

   (b) The star closest to the Earth (other than the Sun) is Proxima Centauri, which has an observed annual parallax of 0.772 arcseconds. How does your answer to the first part compare to the distance to Proxima Centauri? How likely do you think it is that we'll ever see a Type I supernova that appears ten times brighter than the Sun?

4. **(7 points)** Bigger than a Breadbox, Smaller than a Star

   In a paragraph or two, compare and contrast the properties of dwarf planets, planets, and brown dwarfs. (Use Wikipedia, the "Brown Dwarf Article" in the Resources section of bSpace, or any other source freely, but in all cases cite your sources!) What are the characteristics (mass, radius, temperature, distance from a star, density, *etc.*) that distinguish dwarf planets from planets? What are the characteristics that distinguish planets from brown dwarfs? Are there any overlaps between the different categories? Feel free to type up your answer separately and attach it to the rest of your problem set.