Ay 10 - Section Worksheet 4
The Properties of Extrasolar Planets

In this worksheet, you'll see just how much we can learn about extrasolar planets even though it's impossible for us to actually see them! We'll present you with some data about a real star, ε Eridani, and the planet, ε Eridani b, that orbits it. (There are actually *three* planets that orbit ε Eridani, but we will ignore the other ones to make the analysis easier.) First, we'll use what we know about the properties of stars to derive some important basic facts about ε Eridani. Then we'll use some radial velocity (RV) curve data to figure out the mass of the planet. Finally we'll use a transit lightcurve to figure out the physical size of the planet. We can put these pieces of information together to find out how dense ε Eridani b is, which tells us whether it is made out of rock, gas, or a combination of the two.

Properties of the Host Star

First, we'll figure out some properties of ε Eridani. We will do this by using some *scaling relations* given in class and in the textbook. These relations let us figure out facts about ε Eridani, or any other star, based on how it compares to the Sun.

The first scaling relation lets us get the temperature of a star based on the wavelength at which it emits the most photons. Remember that Wien's Law,

$$T_{\rm bb} = \frac{2.9 \times 10^6 \text{nm K}}{\lambda_{\rm peak}},$$

is true for all blackbodies. We can take two versions of Wien's law, one subscripted for the Sun and one for ε Eridani, and divide them by each other, yielding

$$\boxed{\frac{T_{\varepsilon \rm E}}{T_\odot} = \frac{\lambda_{\rm peak,\odot}}{\lambda_{\rm peak,\varepsilon E}}.}$$

Note that the constant in Wien's Law has disappeared! We can apply the same approach to the inverse square law of light, which tells us that the apparent brightness B of an object is its intrinsic luminosity L reduced by the square of our distance from it, d:

$$B = \frac{L}{4\pi d^2}$$

so

$$\boxed{\frac{B_{\varepsilon \rm E}}{B_\odot} = \left(\frac{L_{\varepsilon \rm E}}{L_\odot}\right)\left(\frac{d_\odot}{d_{\varepsilon \rm E}}\right)^2.}$$

The Stefan-Boltzmann law tells us that the intrinsic luminosity of a star is equal to its surface area times the fourth power of its surface temperature:

$$L = \left(4\pi R^2\right)\left(\sigma T^4_{\rm surface}\right)$$

so, dividing once again,

$$\boxed{\frac{L_{\varepsilon\mathrm{E}}}{L_\odot} = \left(\frac{R_{\varepsilon\mathrm{E}}}{R_\odot}\right)^2 \left(\frac{T_{\varepsilon\mathrm{E}}}{T_\odot}\right)^4.}$$

Finally, the luminosity of a star goes roughly as the fourth power of its mass. In math notation,

$$L = kM^4$$

where k is some proportionality constant. Dividing once more, we find

$$\boxed{\frac{L_{\varepsilon\mathrm{E}}}{L_\odot} = \left(\frac{M_{\varepsilon\mathrm{E}}}{M_\odot}\right)^4.}$$

Note that we don't even have to know what the value of k is to get some useful information! This is why "scaling laws", like "luminosity goes as mass to the fourth power", are good – if we have an example that fulfulls the law, we can apply it to any other case easily.

1. The parallax of ε Eridani is 0.311 arcsec. What is its distance from us in parsecs?

2. **Use the boxed equations above** and your answer to the previous question to fill in the blank spots on the following tables:

Star	Apparent Brightness (J / m^2 / s)	λ_{pk} (Å)	Distance (pc)
Sun	1.37×10^2	5000	4.84×10^{-6}
ε Eridani	8.5×10^{-11}	5700	

Star	Temperature (K)	Luminosity ($L_\odot$)	Radius ($R_\odot$)	Mass ($M_\odot$)
Sun	5800	1	1	1
ε Eridani				

The Doppler Method & Radial Velocity Curves

3. Why does a star with a planet orbiting it "wobble"? How can this wobble lead to a Doppler shift (both blueshift **and** redshift) of the light from the star? Draw a picture.

4. Is there a situation where the wobble is relatively large, but we cannot detect the Doppler shift from the star? In this situation, is there another way we can possibly detect the exoplanet?

For the rest of the worksheet we're going to assume that we're viewing the system exactly edge-on (to simplify our analysis), even though this isn't exactly correct for the actual ε Eridani system.

As we saw above, a planet can actually affect the spectrum of the star that it orbits by redshifting and blueshifting its spectrum. We know where the actual frequency peaks for various elements are, so given a spectrum, we can use the Doppler shift equation,

$$\frac{\lambda_{\text{observed}} - \lambda_{\text{actual}}}{\lambda_{\text{actual}}} = \frac{v}{c},$$

to calculate the star's *radial velocity*, v. (This velocity is "radial" because it only measures the star's motion directly towards or away from us, not side-to-side or up-and-down.) The actual analysis is much more sophisticated, but the underlying physical principles are the same.

Astronomers have taken spectra of ε Eridani for about 15 years now, and each time they've analyzed the data to calculate a radial velocity for the star at the time the spectrum was taken. We can make a plot of the date of each spectrum versus the calculated radial velocity of the star at that time. By subtracting off the star's average radial velocity, we can see the wobble caused just by the planet, ignoring the star's overall motion. Doing this produces something that looks like Figure 1. The boxes represent fake "observed" data points and the line represents the mathematical radial velocity curve that agrees with the data the best.

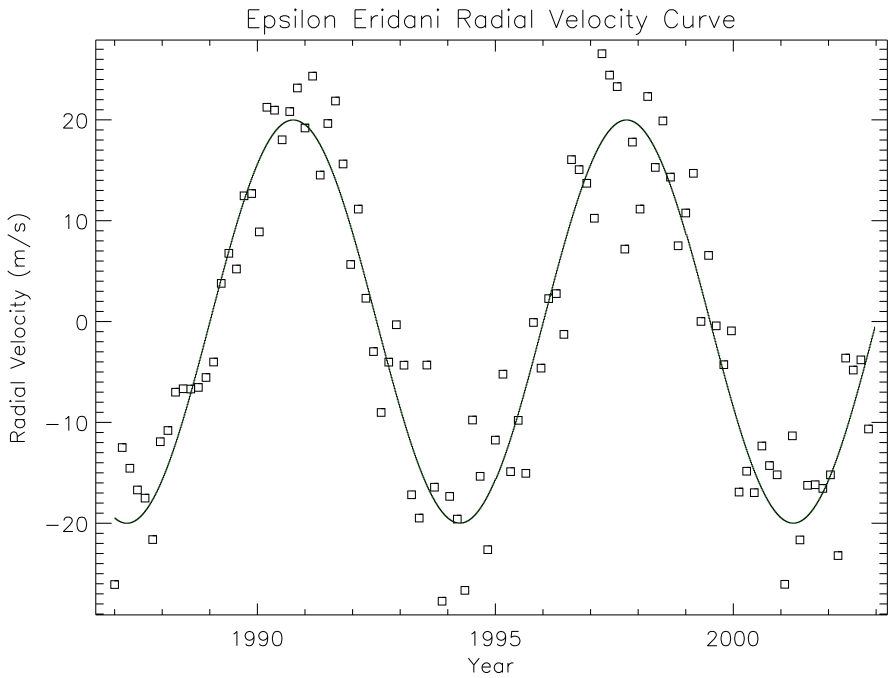

Figure 1: A radial velocity curve

Just from looking at Figure 1 we can figure out a few important things about the ε Eridani system.

5. What is the period of the star's wobble in years? How did you come up with this number? Use the period of the star's wobble to calculate the period of the planet.

6. What is the approximate eccentricity of the planet's orbit? How did you come up with this number?

7. What is the maximum radial velocity with which the star is wobbling in m/s? How did you come up with this number? (Consider: is it more appropriate to use the most extreme single data point, or the most extreme point on the "best fit" curve?)

In order to calculate the semi-major axis of the planet's orbit (i.e. the radius, since we have a circular orbit) we can use Newton's modified version of Kepler's Third Law:

$$P^2 = \frac{a^3}{M_{\text{total}}}$$

The above version of the equation works **only** if the period (the time it takes the planet to make one complete orbit around the star), P, is in years and the semi-major axis (the average distance between the star and the planet), a, is in AU and the total mass of the system (including the star **and** the planet), M_{total}, is in solar masses. We will make the assumption that the mass of the planet, M_p, is much, much smaller than the mass of the star, M_*. In math speak, we're assuming $M_p \ll M_*$. We'll check how good this assumption is shortly.

8. Using the version of Kepler's law shown above, calculate the semi-major axis of the planet orbiting ε Eridani in AU. Be sure to read the paragraph above this one carefully.

Now we have everything we need to calculate the mass of the planet! To do this we will use the equation

$$M_p \approx 0.0353 v_{\text{max}} \sqrt{a M_*}$$

where M_p is the mass of the planet **in Jupiter masses**, v_{max} is the maximum radial velocity of the star in its wobble **in m/s**, a is the semi-major axis of the planet's orbit **in AU**, and M_* is the mass of the star **in solar masses**.[1]

9. What is the mass of the planet orbiting ε Eridani in Jupiter masses? What is its mass in solar masses? Recall that $10^3 M_{\text{Jupiter}} = M_\odot$.

10. Is our assumption that $M_p \ll M_*$ valid?

[1] If you're curious where this equation comes from, I urge you to take Astro 7A: Introduction to Astrophysics I. In that class you'll derive this equation on a homework!

The Transit Method & Transit Lightcurves

Since we're viewing the ε Eridani system exactly edge-on, the planet's orbit is aligned so that it transits in front of ε Eridani when viewed from Earth. Figure 2 below shows a "lightcurve" — that is, the plot of total light received from ε Eridani as a function of time — over the course of one planetary transit. (Just for clarity's sake, this "data" is 100% fake.) As in the previous figure, the crosses show "observed" data points, while the curve shows a mathematical model fit to the data.

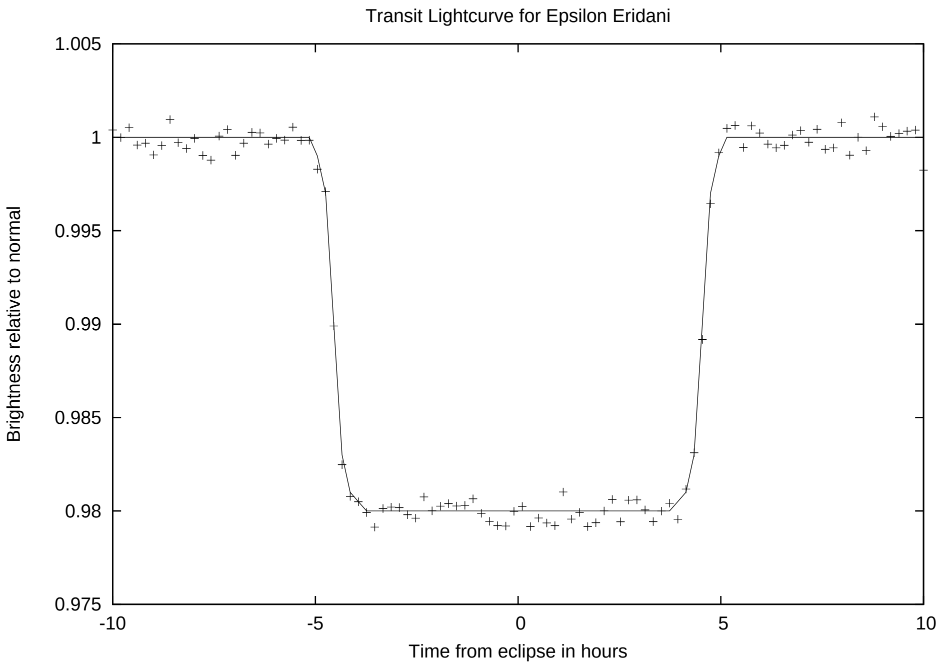

Figure 2: A translit lightcurve

Remember that the amount of light we detect on Earth (the flux) from a distant star is proportional to the surface area of the object that's doing the emitting.

11. When the planet was completely on top of the star, how much of the star's light did we receive? Draw a diagram showing how the planet and star would appear from Earth if we could resolve them.

12. How must the surface area of the planet compare to the surface area of the star? Write down an equation relating the two, using $A_{\varepsilon \mathrm{E}}$ to represent the star's surface area and A_p to represent the planet's surface area.

13. What is the radius of the planet in terms of the radius of ε Eridani?

14. You figured out the radius of the star ε Eridani in the first section. What must the radius of the planet be in Jupiter radii? Note that $10R_{\rm Jupiter} = R_\odot$.

Density, usually represented by the Greek letter rho (ρ), is a term we've used a few times. It's defined as the mass of an object divided by its volume. Recall that the volume of a sphere is $\frac{4}{3}\pi R^3$ where R is the radius of the sphere.

15. You figured out the mass of the planet, ε Eridani b, in the second section. What is its density, in grams per cubic centimeter? Just to remind you again, $M_{\rm Jupiter} = 2 \times 10^{30}$ g and $R_{\rm Jupiter} = 7 \times 10^9$ cm.

16. The density of Earth is about 5.52 g/cm^3, the density of Jupiter is about 1.33 g/cm^3, and the density of water is 1.00 g/cm^3. What is ε Eridani b made of?

17. What planet(s) in our own solar system does it most resemble as far what it's made of?

18. What planet(s) in our own solar system does it most resemble as far as mass and radius?

19. What planet(s) in our own solar system does it most resemble as far as period and semi-major axis?

20. What's the name for this type of exoplanet?