Midterm 1 Review Sheet

Orbital Mechanics

1. Two masses are located at distances r_1 and r_2 from their center of mass.
   (a) Relate r_1 and r_2 to r.
   (b) Find r_1/r_2 in terms of m_1 and m_2.
   (c) Relate a_1 and a_2 to a.
   (d) Find a_1/a_2 in terms of m_1 and m_2.

2. OH NO! The Sun's mass has suddenly decreased by half!
   (a) What is the kinetic energy of the Earth right when the Sun decreases its mass? Express your answer in terms of the reduced mass of the earth-sun system μ, the total mass M and the distance between the reduced mass and total mass r?
   (b) What would be the *new* potential energy of the earth (express your answer in terms of the same parameters given above)
   (c) What would be the total energy of the earth-sun system?
   (d) Using your result, what would happen to the earth in this scenario?

3. Sketch the radial velocity of the Sun due to Jupiter ($a = 5.2$ AU, $M_{jup} = 10^{-3} M_{sun}$) in units of meters per second versus time in years.

4. For each of the types of binary systems below, how would you determine the masses of the components (assume circular orbits)?
   (a) Visual ($i = 0$)
   (b) Eclipsing/Spectroscopic
   (c) Planet with the mass of the star known.

5. Let's calculate a few numbers for a binary star system. Assume $a_1 = 3a_2$. The total mass of the system is $1M_\odot$. The period of this binary orbit is 1 year. Assume circular orbits.
   (a) What's the total semi-major axis (a) of the system?
   (b) What are a_1 and a_2?
   (c) What are m_1 and m_2 (the masses of the individual star)?
   (d) What is the velocity of each star?
   (e) What is the angular momentum of the combined system?
   (f) What is the energy of the combined system?

6. What aspects of the gravitational force law allowed us to write down the virial theorem? Would it work for a different r^{-2} force law? Would it work for a different type of central force law? What about a non-central force law (that means the force depends on more than r)?

Black Body Radiation

1. The visual binary star system Albireo has a yellow star and a blue star. The yellow star is brighter than the blue companion. What's going on here? Can both of these stars be on the main-sequence?

2. Suppose you measure the spectrum, F_λ, of a perfect spherical blackbody and it peaks at some wavelength λ_p. Suppose also that you somehow know its radius, R. Using the spectrum alone (the object is unresolved), determine its distance from us.

3. If stars were perfect blackbodies, what information would we no longer be able to determine through observations?

4. Eliot said in Thursday's lecture that sunspots are cooler areas of the sun's surface. But they are ≈ 4500 K! That's pretty hot. Compare the flux from a sunspot to the flux from the other part's of the sun's photosphere ($T \approx 5700$ K).

5. At what distance would a 100 Watt lightbulb have to be from your hand in order for your hand to receive flux equal to the flux from the sun? (The solar luminosity is 3.9×10^{26} Watts) You can also compare this flux with the blackbody flux emitted from a square cm of your body.

The Spectra of Stars

1. Explain why hydrogen absorption lines vary from one type of star to the next. Make sure to reference the Saha and Boltzmann equations.

2. What's the deal with kT? Pretend you are explaining this to a student who has never seen these equations before. Why is kT important in dealing with the populations of energy levels in an atom?

3. Explain the difference between 'most probable velocity' and 'mean velocity' in a velocity distribution. What assumptions go into using a Maxwell-Boltzmann velocity distribution?

4. For what range of temperatures is partition function Z_I of neutral hydrogen well-approximated by 2?

5. The Boltzmann equation doesn't contain the number density of electrons, n_e. So why does the Saha equation? Why should N_{II}/N_I be inversely proportional to n_e?

6. Estimate a representative velocity of air molecules in this room. HINT: What is the thermal energy in the room? Ignore factors of $\frac{1}{2}$ and stuff.

7. Make a table with one column labeled "What" and the other "How." List the properties of stars we can determine observationally and then next to each property describe how its done.

8. You can argue that there is one fundamental parameter which determines the properties of main-sequence stars. What is that parameter and what are the relations between the other important properties of the star? (here we are ignoring the composition of the star)

Telescopes

1. Suppose you are observing a source which emits light with a spectrum of the form:

$$F_\nu = 3.1831 \times 10^{-23}\nu^3 \text{ ergs/s/Hz/cm}^2$$

   Your telescope has a filter with a frequency response

$$\phi(\nu) = \frac{(1\text{GHz})^2}{\nu^2}$$

   This is just the percentage of photons that are actually detected as a function of frequency. Calculate the total flux recieved if your receiver bandwidth stretches from 1.3 to 1.5 GHz.

2. If you wanted to resolve the solar system from 10 parsecs away, how large of a diameter would you need on your telescope (assume you are observing at visible wavelenghts)?

3. With the VLBA (Very Long Baseline Array), angles on the order of micro-arcseconds can be resolved. Using VLBA, what is the most distant object you could measure a parallax for?

4. Explain why the diffraction limit of an aperture is on the order of lambda/D. HINT: draw a diagram of light rays reaching the two ends of the aperture.