

Final Review Sheet

2-body Mechanics

1. What is the tangential velocity of the Earth as it orbits the Sun?
2. You observe a double-lined spectroscopic binary system. You notice two H_α lines ($\lambda_o = 6564 \text{ \AA}$) and, even though you don't observe a full orbit, you determine that the maximum separation of the lines is 0.328 \AA .
One star is K7 ($0.8 M_\odot$) and the other G0 ($1 M_\odot$). Assume they are in a circular orbit and edge-on ($i = \pi/2$).
 - (a) Derive an expression for the orbital period as a function of relative velocity $v = v_1 + v_2$ and total mass $M = M_1 + M_2$.
 - (b) Calculate the period and semi-major axis of the system.
3. Your friend claims it is hot in the summer because the Earth is closer to the Sun¹. The eccentricity of the Earth's orbit is $e = 0.0167$. Show that the Earth is closer to the Sun by only a factor of $1 - e^2$. (This is an exercise in making good approximations as much as doing 2-body mechanics.)
4. Kepler's Law is often written $P^2 = a^3$. Where does this result come from? What are the units of P and a? What is the constant of proportionality and what are the units of this constant? When can you use this equation?
5. A binary system has a semi-major axis of 2 AU and a period of 2 years. What is the total mass of the system in solar masses?

Radiation

1. Approximately how much cooler is the Earth at aphelion than perihelion due to the eccentricity of the Earth's orbit?
2. Uh oh, it looks like Neil Armstrong littered on the Moon. He left two black boxes by the American flag. Both boxes are cubes with side lengths $\ell_1 = 10\text{cm}$ and $\ell_2 = 100\text{cm}$.
 - (a) Make the approximation that the distance from the Sun to the Moon is 1 AU. How much more flux does box 2 receive during the lunar day than box 1?
 - (b) Compare the energy per second received by the two boxes.
 - (c) Which box gets hotter during the day?
 - (d) What are the equilibrium temperatures of the two boxes in degrees Kelvin?
 - (e) During the lunar night, the boxes try to come into radiative equilibrium with the Universe which has a spectrum that peaks around $\lambda_{max} \approx 0.1\text{cm}$. What are the temperatures of the boxes at night?
 - (f) Something goes terribly wrong one day and the boxes glow red. By how much has the sun's radius increased if it still appears yellow?

¹There are several things wrong with this argument.

Radiative Transfer

1. The radiative transfer equation says:

$$\frac{dF}{dz} = -\rho\kappa F \quad (1)$$

- (a) What does the left side of the equation say? What does it mean?
 - (b) Why is there a negative sign?
 - (c) Why should $\frac{dF}{dz}$ be proportional to ρ ?
 - (d) What is κ ? Why should $\frac{dF}{dz}$ be proportional to κ ?
 - (e) What does it mean for $\frac{dF}{dz}$ to be proportional to F ?
 - (f) Solve for $F(z)$.
 - (g) What is optical depth?
 - (h) What do the terms “optically thin” and “optically thick” mean?
 - (i) Use a Taylor expansion to give the radiative transfer equation in the optically thin regime. What value of τ should you expand about?
2. A slab of ionized hydrogen gas is placed 0.5 AU from the sun. The slab has a thickness $h = 1$ km and has a circular cross-sectional area with radius $b = 100$ km. The opacity κ is due to electron scattering. Recall that $\kappa_{es} = 0.4 \text{ cm}^2 \text{ g}^{-1}$.
 - (a) How much energy per second does the slab of gas receive from the Sun?
 - (b) How much flux does the slab receive from the sun?
 - (c) How would this amount change if the slab had two times more surface area?
 - (d) If the number density is 100 particles per cubic centimeter, what is the flux of scattered radiation?
 3. Estimate how many photons with wavelength 5000\AA the Earth receives per second from the sun.
 4. Under what conditions would a star’s spectrum have no line features?
 5. Under what conditions would a star have no limb brightening?
 6. Are these conditions realistic?
 7. Why is opacity important for stellar structure? What would happen to a star if $\kappa = 0$ throughout its interior?

Stellar Spectra

1. Why do stars of different mass have different spectra?
2. What does the Boltzmann equation describe?
3. What is the difference between the Boltzmann and Saha equations?
4. The Balmer lines ($n=2$ to a higher energy level) occur only for a narrow range of temperatures. Explain why this is. What if the temperature is too low? Too high?

Stellar Structure

1. What are the equations of stellar structure?
2. Translate these equations into English.
3. Estimate the central temperature of the Sun.
4. What is pressure? What kinds of pressure are there? When is each kind important in stellar interiors?
5. Derive the mass-luminosity relationship.
6. Why do high mass stars die so young?
7. What is a density scale height? Derive $n(r)$ for an exponential atmosphere.
8. Derive the scale height for a neutron star's atmosphere.
9. Are all pulsars neutron stars? Are all neutron stars pulsars?
10. What is a Roche lobe? What is a Lagrange point? What do they have to do with stars?
11. Which star likely started its life with more mass, Sirius A or B (B is a WD)?
12. The sun started from a big ball of gas (assume $1 M_{\odot}$ with radius 100 AU) and will end up as a WD. Check to see if energy is conserved.

The Death of Stars

1. What is accretion?
2. Why do accreting objects shine?
3. How can we observationally determine the mass-transfer rate \dot{M} of an accreting object?
4. Compare the differential tidal force experienced by an object of mass m around a) a $4 M_{\odot}$ BH, and b) a $10^6 M_{\odot}$ BH.
5. How can we find black holes?
6. What is degeneracy pressure?
7. Consider a hypothetical new star that is supported by the degeneracy pressure from a new kind of fermion that has a mass $m = 10^{-2} m_n$ (and it only contains this fermion; no protons, no neutrons, etc.). Use scaling arguments to estimate i) the radius of a $1 M_{\odot}$ version of this star, and ii) the maximum possible mass for such a star.
8. One place BHs are found is in "X-ray binaries." Does accretion onto a black hole have to give rise to X-rays? What if the black hole mass is $\gg M_{\odot}$ or the accretion rate is very small?