Ay 7A - Fall 2009
Review Sheet 1
Given the constants and equations sheet, you should feel fairly comfortable solving these problems.

Orbital (2-Body) Mechanics

1. OH NO! The Sun's mass has suddenly been cut in half!
   (a) What is the kinetic energy of the Earth right as the Sun decreases its mass?
   (b) What would be the new potential energy of the Earth (ignoring all other planets?
   (c) What would be the total energy of the Earth-Sun system?
   (d) What would happen to the Earth in this scenario?

2. Sketch the radial velocity of the Sun due to Jupiter ($a = 5.2$ AU, $M_{\rm Jup} = 10^{-3} M_\odot$) in units of cm/s versus time in years.

3. For each of the types of binary systems below, describe how you would determine the masses of the components (assuming circular orbits).
   (a) Visual/astrometric
   (b) Eclipsing
   (c) Spectroscopic (double and single-line)
   (d) Planet with the mass of the star known

4. We have a binary star system with $a_1 = 3a_2$, a total mass of $1M_\odot$, and an orbital period of 1 year. Assume circular orbits.
   (a) What is the total semi-major axis (a) of the system?
   (b) What are a_1 and a_2?
   (c) What are m_1 and m_2 (the masses of the individual stars)?
   (d) What is the velocity of each star?
   (e) What is the angular momentum of the combined system?
   (f) What is the energy of the combined system?

5. Your friend claims it is hot in the summer because the Earth is closer to the Sun.[1] The eccentricity of the Earth's orbit is $e = 0.0167$. Show that the Earth is closer to the Sun by only a factor of 1 - e^2. (This is an exercise in making good approximations as much as doing 2-body mechanics.)[2]

6. Kepler's Law is often written $P^2 = a^3$. Where does this result come from? What are the units of P and a in this version of Kepler's Law? When can you use this equation?

Radiation (Blackbody and Otherwise)

1. Suppose you measure the spectrum, F_λ, of a perfect spherical blackbody and it peaks at some wavelength λ_p. Suppose also that you somehow know its radius, R. Using the spectrum alone (the object is unresolved), determine its distance from us.

2. If stars were perfect blackbodies, what information would we no longer be able to determine through observations?

[1] This statement is wrong for many reasons.
[2] In fact, the Earth is this much closer to the Sun during **winter** in the Northern Hemisphere.

3. Sunspots are cooler areas of the Sun's surface. But they are ~4500 K! That's pretty hot. Compare the flux from a sunspot to the flux from the other parts of the Sun's photosphere (T≈5800 K).

4. At what distance would a 100 Watt light-bulb have to be from your hand in order for your hand to receive flux equal to the flux from the Sun? (The solar luminosity is 3.9×10^{26} Watts.) Also compare this flux with the blackbody flux emitted from a square cm of your body.

5. Approximately how much cooler is the Earth at aphelion than perihelion due to the eccentricity of the Earths orbit?

6. Uh oh, it looks like Neil Armstrong littered on the Moon. He left two black boxes by the American flag. Both boxes are cubes with side lengths $r_1 = 10$ cm and $r_2 = 100$ cm.

   (a) Make the approximation that the distance from the Sun to the Moon is 1 AU. How much more flux does box 2 receive during the lunar day than box 1?

   (b) Compare the energy per second received by the two boxes.

   (c) Which box gets hotter during the day?

   (d) What are the equilibrium temperatures of the two boxes in Kelvin?

   (e) During the lunar night, the boxes try to come into radiative equilibrium with the Universe which has a spectrum that peaks around $\lambda_{\text{peak}} \approx 0.1$ cm. What are the temperatures of the boxes at night?

   (f) Something goes terribly wrong one day and the boxes glow red. By how much has the suns radius increased if it still appears yellow?

The Spectra of Stars

1. Explain why hydrogen absorption lines vary from one type of star to the next. Make sure to reference the Saha and Boltzmann equations.

2. What's the deal with kT? Pretend you are explaining this to a student who has never seen this term before. Why is kT important in dealing with the populations of energy levels in an atom?

3. The Boltzmann equation doesn't contain the number density of electrons, n_e. So why does the Saha equation? Why should $N_{\text{II}}/N_{\text{I}}$ be inversely proportional to n_e?

4. Estimate a representative velocity of air molecules in this room. HINT: What is the thermal energy in the room? Ignore factors of 1/2 and stuff.

Telescopes

1. Suppose you are observing a source which emits light with a spectrum of the form:

   $$F_\nu = 3.1831 \times 10^{-23} \nu^3 \text{ ergs/s/Hz/cm}^2$$

   and your telescope has a filter with a frequency response

   $$\phi(\nu) = \frac{(1 \text{ GHz})^2}{\nu^2}$$

   which is simply the percentage of photons that actually make it to your detector as a function of frequency. Calculate the total flux received if your receiver bandwidth stretches from 1.3 to 1.5 GHz.

2. If you wanted to resolve the Solar System (about 30 AU in radius) at visible wavelengths from 10 pc away, how large of a diameter would you need on your telescope?

3. With the VLBA (Very Long Baseline Array), angles on the order of micro-arcseconds can be resolved. Using the VLBA, what is the most distant object you could measure a parallax for?