Ay 7A – Fall 2009
Section Worksheet 4
Diffraction Action, Get Some Satisfaction

1. **A Two-Star Household**

   Consider a binary star system. Star A has a radius $R_A = 2R_\odot$ and a temperature $T_A = 5000$ K. Star B has a radius $R_B = 0.5R_\odot$ and a temperature $T_B = 10000$ K. The system is located 20 pc from Earth, and the two stars are separated by 5 AU. Suppose that these two stars are in circular orbits around their barycenter (i.e. center of mass) and that we are observing the orbits edge-on.

   Note that except for part (a), you should not have to do any numerical calculations for this problem.

   (a) Assuming you are observing this binary system with the Keck Telescope (10 m diameter) at a wavelength of 5000 Å, could you resolve the binary system? (In other words, could you tell if this was a binary system just by imaging it with Keck?) Ignore any atmospheric effects.

   `The angular resolution (ignoring atmospheric effects) of the Keck telescope at 5000 Å is given by:`

   $$\theta_{\rm res} \approx \frac{\lambda}{D} = (5 \times 10^{-5}\ {\rm cm})/(10^3\ {\rm cm}) \approx 5 \times 10^{-8}\ {\rm radians} \approx 0.01''$$

   `where we recalled that 206265'' = 1 radian.  The angular size of the binary system is given by:`

   $$\theta_{\rm binary} = \frac{r}{d} = (5\ {\rm AU})/(20\ {\rm pc}) \approx 1.2 \times 10^{-6}\ {\rm radians} \approx 0.25''$$

   `Since the angular size of the system is much larger than the resolution limit of Keck, the individual stars` **CAN** `be resolved.`

   (b) If the atmospheric seeing limits resolution to 0.5”, can you still resolve the individual stars?

   `If seeing blurs things to 0.5'', then we` **CANNOT** `resolve the individual stars.  Their separation is too close and they get blurred into one star thanks to the atmosphere.`

   (c) Draw (qualitatively) the lightcurve (i.e. flux observed versus time) of one full period of the *total* system. In other words, what is the combined flux of this system versus time. You can assume here that flux here means “bolometric flux” (i.e. integrated over all wavelengths).

   `The lightcurve should look something like Figure 7.9 in Carroll and Ostlie (p. 190). The flux is basically constant for most of the time. It drops some amount as Star B is being eclipsed by Star A, but then stays constant while Star B is behind Star A. It then rises again as Star B comes out from behind Star A, stays constant, then drops again as Star B moves in front of Star A. It stays constant at this lower level as Star B eclipses part of Star A, then rises again as Star B moves out from in front of Star A, and finally it returns to the highest constant level. This sequence will repeat indefinitely, even though you only need to draw one period.`

   (d) If this system were seen face-on instead of edge-on, what would the lightcurve look like?

   `A face-on system would never eclipse, therefore the lightcurve would be constant (and would equal the combined light of both stars).`

(e) In the edge-on lightcurve, there should be two dips in brightness for every orbital period (make sure you understand why!). Which dip is bigger (i.e. larger drop in observed brightness) or are they equal in size?

Essentially the same area is being obscured whether the small star is in front or behind the bigger star, so the only thing that matters is the relative flux of the two stars (which is dependent only on temperature through the Stefan-Boltzmann Law!!). Thus the larger dip occurs when the hotter star is eclipsed (i.e. behind) the cooler one, because the greater flux between the two is being obscured. In our scenario, the bigger dip would then be when Star B is eclipsed by Star A.

(f) What if Star B had $T_B = 2500$ K?

Star B is now colder than Star A so the bigger dip in brightness occurs when Star B passes *in front* of Star A (i.e. when Star A is partially eclipsed by Star B).

(g) What if Star B had $T_B = 5000$ K?

Equal temperatures mean equal sized dips in brightness.

2. **Radio Telescopes vs. Mosquitoes**

The Very Large Array (VLA) in Socorro, New Mexico is a 27 dish radio telescope array. Each dish is 25 meters in diameter. The VLA has been operating since 1976.

(a) Assume that the VLA has been continuously observing a source with a flux density of 1 Jansky (Jy) using a bandwidth of 10 MHz since 1976 and that it is 100% efficient. Estimate how much total energy it has detected in that time. (Recall that 1 Jy = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}.)

According to your reading, the total power (energy per second, or luminosity) collected by a radio telescope is:

$$P = \int_A \int_\nu S_\nu(\nu) f_\nu d\nu dA$$

where A is the collecting area. Here, $S_\nu(\nu)$ is constant over the bandwidth and $f_\nu = 1$. This means that the equation becomes $P = SA\Delta\nu$. The collecting area is:

$$\pi(1250 \text{ cm})^2 \times 27 = 1.3 \times 10^8 \text{ cm}^2$$

and so

$$P = (1 \text{ Jy})(1.3 \times 10^8 \text{ cm}^2)(10 \times 10^6 \text{ Hz}) = 1.3 \times 10^{-8} \text{ erg/s}$$

If the VLA has been observing this source continuosly since 1976 (33 years = 1.04×10^9 seconds) the total energy detected would be about 13 ergs.

(b) Estimate how much energy is expended by a mosquito doing 1 push-up. How many push-ups does the mosquito need to do to match the total amount of energy detected by the VLA?

The energy expended by a mosquito doing a pushup will basically be the work done during the pushup. The work done will basically be the change in gravitational potential energy $\Delta U = mg\Delta h$. We can estimate that a mosquito is about 1 mg (= 10^{-3} g) and that it moves about 1 mm (= 0.1 cm) up during the pushup. The energy expended by the mosquito is:

$$U = (10^{-3} \text{ g})(980 \text{ cm/s}^2)(0.1 \text{ cm}) = 0.1 \text{ erg}$$

This means that it would take a mosquito about 130 pushups to equal the total amount of energy detected by the VLA.

(c) (**OPTIONAL**) The VLA can be put into many different configurations which achieve different angular resolutions. In the "A configuration" the maximum separation between the dishes is 36 kilometers. What is the smallest angle that can be resolved by the VLA in this configuration at a frequency of 1 GHz?

`The smallest angle that can be resolved is equivalent to the diffraction limit of the VLA. We can find this using` $\theta \approx \lambda/D$. `Here:`

$$\lambda = \frac{c}{1\ \text{GHz}} = \frac{3 \times 10^{10}\ \text{cm/s}}{10^9\ \text{Hz}} = 30\ \text{cm}$$

`Plugging this in to the equation we find:`

$$\theta = \frac{30\ \text{cm}}{36 \times 10^5\ \text{cm}} = 8.3 \times 10^{-6}\ \text{radians} \approx 1.7"$$

3. **How Fat is Your Hair**[1]

Even though we approximate many astrophysical sources (such as stars) as point sources, we know that quantum mechanics won't allow nature to let us observe *exact* point sources. The *diffraction limit* (as referred to in the above problems) is the absolute smallest angle that a telescope can resolve, even under perfect atmospheric conditions.

Recall that the diffraction limit comes from the fact that incoming, parallel light rays from a source diffract when they enter the telescope aperture and cause destructive and constructive interference. Thus the light in your telescope is smeared out in a series of maxima and minima, with the majority of light contained within the first maximum.

In lecture we saw pictures of these so-called Airy Discs or Rings and learned that the pattern can be described by:

$$\sin\theta_m = \frac{m\lambda}{d}$$

where d is the diameter of your diffraction slit (in most cases the diameter of your telescope), λ is the wavelength of the light you are observing, m is the order of the minimum (always an integer), and θ_m is the angle between the principle maximum and m^{th} minimum.

Using the principle of diffraction (along with a laser pointer and a ruler that measures lengths no smaller than 1 cm) we are now going to measure the thickness of your hair. First acquire a hair (please don't pick one up off the ground, that's gross!) by having one person in your group gently run a hand through his or her hair. At least one hair should easily remove itself. That person will hold the hair throughout the experiment.

QUICK LASER SAFETY NOTE: Lasers are incredible devices that produce a stream of coherent photons at a single wavelength. Today we will be working with red laser pointers which have $\lambda \approx 7250$Å. As a coherent light source lasers can be extremely dangerous and should not be treated as a toy. Do not under any circumstances shine the laser at another person, especially another person's face. Even with these lasers, considerable damage could be done to someone's eyes.

(a) Have someone who's *not* holding the hair place the laser on a table a couple meters or so from a wall and point the laser toward the wall. Record the distance from the laser to the wall.

`140 cm`

[1]Thanks to Chat Hull and Adam Miller for coming up with this activity.

(b) Now, have the person holding the hair slowly pass the hair through the laser beam just in front of the laser pointer. Write a brief description of any changes you see on the wall.

`When the hair passes through the laser beam it acts as a slit that diffracts the incoming light and thus we see the standard interference pattern from a single slit on the wall.`

(c) While the hair is in the laser beam, have a third person measure the distance between the principle maximum and the first minimum of the pattern on the wall.

`1.0 cm`

(d) Use the information you've gathered (and that has been given to you) in this problem to calculate the width of your hair.

Clearly, the angle θ_m is small so we can use small angle approximations and thus:

$$\sin\theta_m \approx \theta_m = 1.0/140 \approx 0.0071 \text{ radians}$$

where we have plugged in the values that we measured in parts (a) and (c). Note that to be technically accurate, though it is a small correction, we should measure the angle from the slit (i.e. the hair), NOT from the front of the laser beam. However if you put the hair right against the laser pointer, then they're the same distance effectively. Since we measured the distance from the principle maximum to the **first** minimum, we know that $m = 1$ and we are told that the wavelength of the red laser pointers are ~ 7250Å, thus:

$$\sin\theta_m \approx 0.0071 = \frac{(1) \times (7250 \text{ Å})}{d}$$

where d is the thickness of the hair (in Å). Solving for d, I get that the thickness of the hair is about 100 microns. According to the Interwebs, the average human hair is between 20 and 200 microns, with darker hair slightly thicker than lighter hair. Finally, note that in order for the diffraction equation to be valid we need to have the wall ''very far away'' (i.e. the distance to the wall must be much larger than the width of the slit -- which it obviously is).