Ay 7A - Fall 2009
Section Worksheet 2

1. **Escape Velocity**

   (a) A planet is orbiting around a star. What is the total orbital energy of the planet? (i.e. Total Energy = Potential Energy + Kinetic Energy). To just barely escape the orbit (and the gravitational attraction of the star), what is the minimum total energy the planet should have?

   (b) Using the above result, what is the escape velocity of the planet from the star? (The **escape velocity** is defined as the minimum velocity an object needs to escape the gravitational pull of another object.)

   (c) Calculate the escape velocity of an object from the surface of the Earth (only considering the Earth's gravitational pull).

2. **Geosynchronous Satellite**

   (a) Communications and weather satellites are often placed in geosynchronous "parking" orbits above earth. These are orbits where satellites can remain fixed above a specific point on the surface of Earh. At what altitude must these satellites be located?

   (b) Is it possible for a satellite in a geosynchronous orbit to remain "parked" over any location on the surface of Earth? Why or why not?

(c) Jeff has DirecTV at his house in the Berkeley Hills ($\sim 38°$ north latitude). How many degrees away from the zenith (which is what astronomers call "directly overhead") should he point his satellite dish in order to be aimed at the nearest place a geosynchronous satellite could be?

3. **Hohmann Transfer**

Orbits are changed by single or multiple thrusts of the rocket engines. The simplest maneuver is a single thrust applied in the orbital plane that does not change the direction of the angular momentum but does change the eccentricity and energy simultaneously. The most economical method of interplanetary transfer consists of moving from one circular heliocentric (sun-oriented motion) orbit to another in the same plane. Earth and Mars represent such a system reasonably well, and a Hohmann transfer represents the path of minimum total energy expenditure. Two engine burns are required: (1) the first burn injects the spacecraft from the circular Earth orbit to an elliptical transfer orbit that intersects Mars' orbit; (2) the second burn transfers the spacecraft from the elliptical orbit into Mars' orbit. We will calculate the velocity changes needed for a Hohmann transfer in this problem. Here we are considering only the gravitational attraction of the sun and not that of Earth and Mars (Figure 1). For this problem you don't have to plug in numbers if you don't want to (until the last part), but you should be able to solve everything in terms of the mass of the Sun, the Sun-Earth distance, the Sun-Mars distance, and the gravitational constant, G.

(a) The spacecraft is moving with $\mathbf{v_1}$ in the orbit of the Earth around the Sun (the Sun-Earth distance is r_1). What is the velocity needed to "kick" it into an elliptical transfer orbit that can reach Mars' orbit (the Sun-Mars distance is r_2)? You may start with the total orbital energy equation you used in 'Escape Velocity' problem.

(b) Similarly, for the transfer from the ellipse to the circular orbit of radius r_2, what is the transfer speed needed?

(c) What is the total time required to make the transfer?

(d) Plug in the appropriate constants and calculate the time needed for a spacecraft to make a Hohmann transfer from Earth to Mars. Assume both planets are in coplanar orbits.

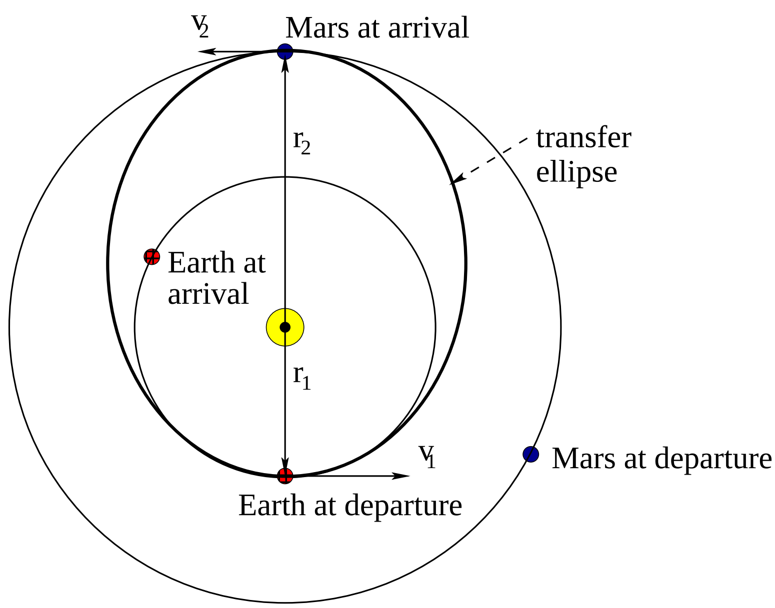

Figure 1: A diagram of Hohmann Transfer

4. **(optional) Homogeneous Sphere**

(a) Imagine a sphere has constant density ρ. What is the mass of the sphere (as a function of its radius, r)?

(b) Using the Newton's first[1] and second theorem[2], get the equation of the motion in terms of ρ and r if a test particle is released from rest at radius r in the gravitational field of a homogeneous sphere.

(c) Using Cartesian coordinates (x, y) defined by $x = r\cos\theta$, $y = r\sin\theta$ and defining $\Omega^2 = \frac{4\pi G\rho}{3}$, rewrite the equations of motion of x and y in terms of Ω.

(d) What is the form of the equations of motion? Write down the general solution of the equations of motion.

(e) What is the shape of the orbits of the test particle? Are the orbits closed? If we assume the mass is concentrated in a point, where is the point located? Compare this to Keplerian motion we derived in class[3].

[1] Newton's first theorem: A body that is inside a spherical shell of matter experiences no net gravitational force from that shell.

[2] Newton's second theorem: The gravitational force on a body that lies outside a spherical shell of matter is the same as it would be if all the shell's matter were concentrated into a point at its center.

[3] Note that Keplerian motion refers to motion in the gravitational field due to a point mass M corresponding to the potential $-\frac{GM}{r}$.