Ay 7A – Fall 2009
Section Worksheet 9
And I'm Free, Free Fallin' – The Jeans Criterion

Stars are born from the gravitational collapse of clouds of interstellar gas. But what determines whether or not a given cloud will collapse? Why aren't all clouds of gas in the interstellar medium (ISM) of the Galaxy undergoing gravitational collapse and forming stars? In this problem we will examine the necessary conditions for collapse to occur.

1. If we have a cloud of gas of mass M and initial mass density ρ, what is the cloud's average potential energy, $\langle U \rangle$, in terms of the variables given and fundamental constants?

$$\langle U \rangle \approx -\frac{GM^2}{R} \approx -GM^{(5/3)}\rho^{(1/3)}$$

`where we have used the fact that` $\rho \approx M/R^3$.

2. In a gas cloud, the kinetic energy is due to the random motions of the N particles that make up the cloud. Our cloud has a temperature T and a mean molecular weight μ. Estimate the average kinetic energy, $\langle K \rangle$, of the cloud in terms of the variables given and fundamental constants.

$$\langle K \rangle \approx N\frac{1}{2}\mu m_{\mathrm{H}} v_{\mathrm{th}}^2$$

`The thermal velocity,` v_{th}`, is given by`

$$v_{\mathrm{th}} \approx \sqrt{\frac{kT}{\mu m_{\mathrm{H}}}}$$

`and the number of particles,` N`, is given by`

$$N = \frac{M}{\mu m_{\mathrm{H}}}$$

`Putting all this together we get`

$$\langle K \rangle \approx \frac{M}{\mu m_{\mathrm{H}}}\frac{1}{2}\mu m_{\mathrm{H}}\frac{kT}{\mu m_{\mathrm{H}}}$$

$$\langle K \rangle \approx \frac{MkT}{2\mu m_{\mathrm{H}}}$$

3. Recall the Virial Theorem:

$$\langle U \rangle = -2\langle K \rangle$$

If this equality holds, we say the system is in "virial equilibrium". However, if the force of gravity in the cloud is greater than the gas pressure force (i.e., the kinetic energy), the system will collapse. Thus collapse will occur if

$$|\langle U \rangle| > |-2\langle K \rangle|$$

Plug your expressions for $\langle U \rangle$ and $\langle K \rangle$ into this inequality and solve for mass (and call it M_J).

$$|-GM_J^{(5/3)}\rho^{(1/3)}| > |-2\frac{M_J kT}{2\mu m_{\rm H}}|$$

$$GM_J^{(5/3)}\rho^{(1/3)} > \frac{M_J kT}{\mu m_{\rm H}}$$

$$GM_J^{(2/3)}\rho^{(1/3)} > \frac{kT}{\mu m_{\rm H}}$$

$$M_J > \left(\frac{kT}{G\mu m_{\rm H}\rho^{(1/3)}}\right)^{(3/2)}$$

4. The critical mass you derived above, the **Jeans mass**[1], is the minimum mass needed for a cloud of gas to undergo spontaneous gravitational collapse. Typical diffuse clouds in the ISM have masses ranging from 1 to 100 $M_\odot$. They have typical temperatures and number densities of $T \sim 100$ K and $n \sim 50$ cm^{-3}. Assuming that such clouds are composed entirely of atomic hydrogen, would you expect them to be sites of star formation?

   Pure atomic hydrogen means $\mu = 1$. Also, we're given n, but we need ρ. Luckily we know that $\rho = n\mu m_{\rm H} \approx 8.35 \times 10^{-23}$ g cm^{-3}. Plugging in the numbers given we get that $M_J \approx 2400$ $M_\odot$. Thus the Jeans mass for diffuse clouds in the ISM is **waaaaay** bigger than the actual observed cloud masses so we **do not** expect these clouds to undergo gravitational collapse and form stars.

[1] Named after the British physicist Sir James Jeans who was the first to consider this problem.