Ay 7A - Fall 2009
Section Worksheet 1

1. **Order of Magnitude** Astronomers have been able to "weigh" the mass of visible stars in the Solar neighborhood—roughly a 100 pc cube centered on the Sun. The value is a little less than 0.05 $M_{\odot}\text{pc}^{-3}$. If other areas in the Milky Way are similar to our neighborhood, about how many stars are in the Galaxy? Is this a good assumption? What other assumptions go into making this estimate?

2. **How Yookyung and Jeff remember the units of physical constants** To be honest, we GSIs have a bad memory. We can never remember the units of constants such as the universal gravitational constant, G. But instead of always looking it up in a book, we can figure it out quickly from simple equations and by remembering a few units from Physics 7A, like position, mass and time. In cgs, these are cm, g and s respectively.

   (a) Remember what Newton said about gravitation, namely:

   $$\vec{F} = \frac{GMm}{r^2}\hat{r}$$

   Use this equation along with Newton's second law to give the units of force in cgs units and then show how this can be used to find the units of the constant G. (NOTE: don't give a number, just units)

(b) Using a method similar to the one above, figure out the units of the Stefan-Boltzmann constant σ, using the following equation for the luminosity of a star (where r is the radius and T the temperature). You'll use this law a lot later…

$$L = 4\pi r^2 \sigma T^4$$

NOTE: Luminosity has the units of power (or energy per unit time).

3. **Dimensional Analysis** This is one of the most important skills you should have as a scientist! In the real world, when astronomers are trying to figure out how things work, they often don't initially care much about factors of 3 or $\frac{1}{2}$ (you will notice as you study physics that constants are usually of order unity, so this isn't a bad first guess). Instead, they sometimes take a quick and dirty crack at the equation they are looking for by just doing something called dimensional analysis. If you know a quantity depends only on a few variables, you can rearrange them (multiply, divide, etc...) so that the units work out.

   Figure out the Schwarzschild Radius: The radius from which light can just escape a black hole, also known as the Point of No Return. The Schwarzschild Radius depends only on G, M (black hole mass), and the speed of light c.

4. **"Is this the right answer?"** GSIs get this question a lot from students. There are several ways you can answer this question for yourself. The first way is by checking the units. Do they match on both sides of the equal sign?

   The second way is to see if things "scale" properly. If I increase the mass of an object, how does it affect the object's acceleration due to a force F? If I decrease the speed, what should happen to the time it takes to travel a given distance? These are the questions you should ask yourself after you get a mathematical result.

   Check these answers:

(a) The distance a projectile flies when fired across a level field.

$$d = \frac{2v^2 \cos\theta \sin\theta}{g}$$

θ is the angle the shot is fired at with respect to the ground.

(b) The central pressure P_c of a star scales roughly as

$$P_c \sim GM\rho R$$

where G is the gravitational constant, M the mass of the star, R its radius and ρ its density.

5. **How to Measure The AU** At the first quarter of the Moon (position Q), the angle EQS is a right angle (see Figure 1).

(a) Convince yourself that the two angles labeled β are in fact equal.

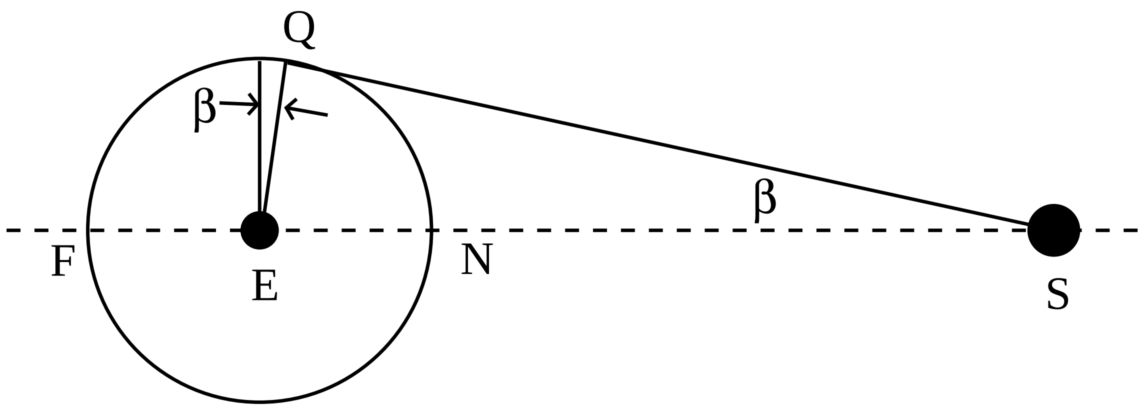

Figure 1: Geometry of lunar phases. The point Q corresponds to first quarter, because the angle EQS is exactly 90 degrees.

(b) Modern observations show that the interval from new Moon (near position N) to first quarter (Q) is 35 minutes shorter than from first quarter to full Moon (near F). Given that the lunar synodic period (the interval between two identical lunar phases) is 29 days and 12.73 hours, estimate the Earth-Sun distance, otherwise known as the Astronomical Unit, in terms of the Earth-Moon distance.

(c) Given that the average Earth-Moon distance is 3.84×10^{10}cm, what is the AU in cm?

(d) How does thisnumber compare to the actual number of cm in AU, 1.50×10^{13}cm?