Ay 7A - Fall 2009
Section Worksheet 1

1. **Order of Magnitude** Astronomers have been able to "weigh" the mass of visible stars in the Solar neighborhood—roughly a 100 pc cube centered on the Sun. The value is a little less than 0.05 $M_\odot \mathrm{pc}^{-3}$. If other areas in the Milky Way are similar to our neighborhood, about how many stars are in the Galaxy? Is this a good assumption? What other assumptions go into making this estimate?

   We have to assume that the average star in the Galaxy has the mass of our Sun, so the average stellar number density of the Galaxy is just 0.05pc^3. We then assume that the visible stars are distributed homogenously in a sphere of size 10kpc. Of course, the stellar distribution is more complicated, with most visible stars confined to the disk; globular clusters in the halo; and a higher number density toward the bulge. But this spherically homogeneous distribution is good enough our order of magnitude purposes. Thus, the volume of the visible galaxy is roughly 10^{12}pc^3. Multiply this by the number density, which gives us 5×10^{10} stars. This is pretty close to the true value of 10^{11} stars!

2. **How Yookyung and Jeff remember the units of physical constants** To be honest, we GSIs have a bad memory. We can never remember the units of constants such as the universal gravitational constant, G. But instead of always looking it up in a book, we can figure it out quickly from simple equations and by remembering a few units from Physics 7A, like position, mass and time. In cgs, these are cm, g, and s respectively.

   (a) Remember what Newton said about gravitation, namely:

   $$\vec{F} = \frac{GMm}{r^2}\hat{r}$$

   Use this equation along with Newton's second law to give the units of force in cgs units and then show how this can be used to find the units of the constant G. (NOTE: don't give a number, just units)

   Newtons second law tells us that $\vec{F} = m\vec{a}$ so we know the units of force in cgs are [g][cm][s]$^{-2}$. Now lets rearrange the gravity equation and figure out the units of G.

   $$[G] = \left[\frac{F\,r^2}{M\,m}\right] = \frac{g\;cm\;s^{-2}\;cm^2}{g^2}$$

   From this we can see that the units of G are cm^3/(g s^2).

   (b) Using a method similar to the one above, figure out the units of the Stefan-Boltzmann constant σ, using the following equation for the luminosity of a star (where r is the radius and T the temperature). You'll use this law a lot later...

   $$L = 4\pi r^2 \sigma T^4$$

   NOTE: Luminosity has the units of power (or energy per unit time).

   The units of luminosity as mentioned above are energy per time. There are many ways to remember the units of energy, for example we know that kinetic energy is $\frac{1}{2}mv^2$ which would have units of g cm^2 s^2. Now we can rearrange the equation to find the units of σ.

$$[\sigma] = \left[\frac{L}{r^2\,T^4}\right] = \frac{g\;cm^2}{s^2\;s\;cm^2\;K^4}$$

`Thus, the units of` σ `are g/(`s^3 `` `$\mathrm{K}^4$`).`

3. **Dimensional Analysis** This is one of the most important skills you should have as a scientist! In the real world, when astronomers are trying to figure out how things work, they often don't initially care much about factors of 3 or $\frac{1}{2}$ (you will notice as you study physics that constants are usually of order unity, so this isn't a bad first guess). Instead, they sometimes take a quick and dirty crack at the equation they are looking for by just doing something called dimensional analysis. If you know a quantity depends only on a few variables, you can rearrange them (multiply, divide, etc...) so that the units work out.

   Figure out the Schwarzschild Radius: The radius from which light can just escape a black hole, also known as the Point of No Return. The Schwarzschild Radius depends only on G, M (black hole mass), and the speed of light c.

   Start by recalling the units of these quantites. The Schwarzschild radius will be a length and have units of cm. The units of G, as we just calculated, are cm^3/(g s^2). Finally, the mass will be in g and the speed of light is cm/s. Since grams do not appear in the answer, G and M must be multiplied together. The only way, using the remaining units, to get rid of s^2 is to divide GM by c^2:

$$R_S \sim \frac{G\,M}{c^2}$$

4. **"Is this the right answer?"** GSIs get this question a lot from students. There are several ways you can answer this question for yourself. The first way is by checking the units. Do they match on both sides of the equal sign?

   The second way is to see if things "scale" properly. If I increase the mass of an object, how does it affect the object's acceleration due to a force F? If I decrease the speed, what should happen to the time of travel? These are the questions you should ask yourself after you get a mathematical result.

   Check these answers:

   (a) The distance a projectile flies when fired across a level field.

$$d = \frac{2v^2\cos\theta\sin\theta}{g}$$

   θ is the angle the shot is fired at with respect to the ground.

   First lets look at the units. d will be in cm on the left-hand side of the equation. On the right-hand side, $\cos\theta$ and $\sin\theta$ have no units, v^2 has units of (cm/s)^2 and g has units of cm/s^2. Dividing v^2 by g gives you units of cm, so the units are correct. Now lets check the scaling. If you increase gravity, you would expect the distance travelled by the projectile to decrease. This is true of the equation. If you increased the velocity you would expect the distance travelled to increase as well, and this is also true. The equation looks good and is, in fact, correct.

(b) The central pressure P_c of a star scales roughly as

$$P_c \sim GM\rho R$$

where G is the gravitational constant, M the mass of the star, R its radius and ρ its density.

`Check the units. Pressure is force per unit area:`

$$[P_c] = \frac{g\ cm/s^2}{cm^2} = \frac{g}{cm\ s^2}$$

$$[GM\rho R] = \frac{cm^3}{g\ s^2} g \frac{g}{cm^3} cm = \frac{g\ cm}{s^2}$$

`The units of this equation dont work out. Lets figure out why with the scaling. If we increase the mass of the planet leaving everything else the same, we expect the central pressure to increase. This is true of the equation. If we increase the density, we expect the central pressure to increase. Also true. If we increase the radius we expect the central pressure to decrease. There is the problem.` $R$ `needs to be on the bottom of the fraction and then the equation is correct.`

5. **How to Measure The AU** At the first quarter of the Moon (position Q), the angle EQS is a right angle (see Figure 1).

(a) Convince yourself that the two angles labeled β are in fact equal.

`Angle QES is` $\alpha = \beta - 90^\circ$. `The line ES is perpendicular to the vertical line from E to the Moon's orbit. Thus that small angle (QE-vertical line) plus` α `equals` 90°. `Thus that small angle must be` β.

(b) Modern observations show that the interval from new Moon (near position N) to first quarter (Q) is 35 minutes shorter than from first quarter to full Moon (near F). Given that the lunar synodic period (the interval between two identical lunar phases) is 29 days and 12.73 hours, estimate the Earth-Sun distance, otherwise known as the Astronomical Unit, in terms of the Earth-Moon distance.

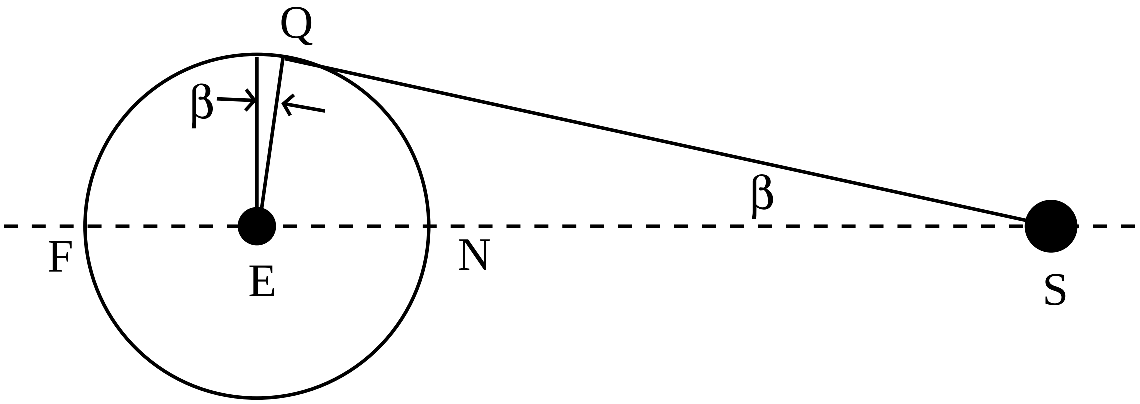

Figure 1: Geometry of lunar phases. The point Q corresponds to first quarter, because the angle EQS is exactly 90 degrees.