# Ay 7A – Fall 2009
# Section Worksheet 8
# The Answer, My Friend, is Blowin' in the (Stellar) Wind

1. **Eddington Luminosity**

   The Eddington Luminosity (or Eddington Limit — named after Sir Arthur Stanley Eddington) is the maximum luminosity a star can have before mass starts getting blown off the star's surface due to radiation pressure. In this problem, will derive an OOM expression for the Eddington Luminosity.

   (a) As we saw in lecture, the equation for radiation pressure is

   $$P_{\rm rad} = \frac{a}{3}T^4$$

   where $a$ is the so-called "radiation constant" which is equal to $4\sigma/c$. Rewrite the radiation pressure equation in terms of the flux of a star. (HINT: As usual, assume the star emits like a black body.)

   `For a black body we know that`

   $$F = \sigma T^4$$

   `So,`

   $$P_{\rm rad} = \frac{a}{3}T^4 = \frac{4\sigma}{3c}T^4 = \frac{4}{3c}F$$

   (b) Rewrite the radiation pressure equation in terms of the luminosity of a star.

   `We know that`

   $$L = 4\pi R^2 F$$

   `So,`

   $$P_{\rm rad} = \frac{4}{3c}F = \frac{4}{3c}\frac{L}{4\pi R^2} = \frac{L}{3\pi c R^2}$$

   (c) Take the derivative with respect to the star's radius of your equation. This now tells us how radiation pressure changes with radius.

   $$\frac{dP_{\rm rad}}{dr} = \frac{d}{dr}\frac{L}{3\pi c R^2} \approx \frac{-2L}{3\pi c R^3}$$

   (d) Also recall from lecture our equation for hydrostatic equilibrium (HSE)

   $$\frac{dP}{dr} = -\rho g$$

   Set this equation for pressure balancing gravity equal to our equation for radiation pressure and solve for $L$.

   $$-\rho g \approx \frac{-2L}{3\pi c R^3}$$

   $$L \approx \frac{3\pi c R^3 \rho g}{2}$$

   (e) We can clean this equation up a bit by replacing $g$ with our usual OOM expression for the gravitational acceleration (i.e., $g \sim GM/R^2$). Go ahead and make this substitution into our luminosity equation.

   $$L \approx \frac{3\pi c}{2}R^3\rho\frac{GM}{R^2} = \frac{3\pi c}{2}R\rho GM$$

(f) Also, since we want to derive the luminosity at which radiation pressure starts blowing mass off the surface of the star, we can use the fact that we're working at the surface to say that

$$\tau = 1 = n\sigma R = \frac{\rho}{m_p}\sigma_T R$$

$$R = \frac{m_p}{\rho\sigma_T}$$

where the number density is simply the mass density divided by the mass of the proton (since stars are mostly ionized hydrogen and thus the protons are making up the bulk of the mass density) and the relevant cross-section for a stellar photosphere we have approximated to be the Thomson cross-section of an electron ($6.65 \times 10^{-25}$ cm$^2$).

Make this replacement for $R$ in our luminosity equation. This is now the equation for the Eddington Luminosity (good to within a factor of 3).

$$L \approx \frac{3\pi c}{2}\frac{m_p}{\rho\sigma_T}\rho G M$$

$$L_{\text{Edd}} \approx \frac{3\pi c G M m_p}{2\sigma_T}$$

2. **How Fat is Fat?**

Astronomers have measured the luminosity of many stars and the most luminous "main-sequence" stars (i.e., stars that are fusing hydrogen to helium in their cores) are found to be about $10^6$ times more luminous than the Sun. As we'll soon see in class, $L \propto M^3$, which means that the most massive stars will also be the most luminous **AND** the most massive stars will be shining at their Eddington Luminosity. Using this information, derive the approximate mass of the most massive stars (in solar masses).[1]

Some useful constants are:

$c = 3.00\times 10^{10}$ cm/s

$G = 6.67 \times 10^{-8}$ dyne cm$^2$/g$^2$

$m_p = 1.673 \times 10^{-24}$ g

$M_\odot = 2.0 \times 10^{33}$ g

$L_\odot = 3.90 \times 10^{33}$ erg/s

`Plugging in the numbers we get`

$$(10^6)(4 \times 10^{33}) \approx \frac{3 \times 3}{2} \times \frac{(3 \times 10^{10})(7 \times 10^{-8})(2 \times 10^{-24})}{7 \times 10^{-25}} M$$

$$4 \times 10^{39} \approx 30 \times 10^3 M = 3 \times 10^4 M$$

$$M \approx 1.3 \times 10^{35} \text{ g} \approx 100 \text{ M}_\odot$$

`Thus, the maximum mass a main-sequence star can have is about 100` $\text{M}_\odot$.

[1] This OOM calculation turns out to be pretty good, but we are a bit lucky to get so close to the observed right answer. There are quite a few considerations we ignored when thinking about the maximum mass of a star and in fact the full answer as to what sets this maximum mass is still an open question!