Ay 7A – Fall 2009
Section Worksheet 5
The Serious Science of Studying Solar Spectra[1]

In this worksheet we are going to find the relative abundance of calcium in the Sun's atmosphere by looking at the solar spectrum. This is a very important technique to understand because it gives us most of the information we know about the composition of stars and other astronomical objects.
If you look at the "strongest" absorption lines in the spectrum of the Sun, one thing you may notice is that the CaII H & K lines are stronger than the Hα line ("stronger" in this context means a deeper and wider absorption line). Check out Figure 1 to see these lines and notice that they are relatively close in wavelength (i.e. energy). A typical assumption here might be that there is more calcium than hydrogen in the Sun. However, I expect all of you already know that there is much more hydrogen than calcium in the Sun. If you didn't know that, you should after doing this worksheet! Now we're going to explore some of the factors that enter into how strong a spectral line is.

1. Calculate the energy in eV of the ground state and first two excited states of hydrogen. Try to remember how to do this without looking up the equation. (Hint: the ionization energy of hydrogen is 13.6 eV, a number which you might want to memorize if you are going into astronomy.) The Hα line is the transition from the n=2 to the n=3. What is ΔE for this transition?

2. In order to find out the abundance of calcium relative to hydrogen we need to compare the strengths of their spectral lines. Let's start with Hα, and think about what determines the strength of this line. Assume that the hydrogen is essentially all neutral, i.e. not ionized, which in astronomy lingo we would write as HI. The effective temperature of the sun is 5770 K.

   (a) What is the average energy of an atom in the Sun in eV? Recall that $k = 1.38 \times 10^{-16}$ erg/K and 1 eV is about 1.60×10^{-12} erg.

   (b) The determining factor here is going to be the fraction of hydrogen atoms with an electron in the first excited state (since these are the only hydrogen atoms that can emit Hα light) to the total number of hydrogen atoms. Calculate this fraction using Boltzmann's equation:

   $$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

   where N_1 is the number of (neutral) hydrogen atoms with its electron in the n=1 (ground) state and N_2 is the number of (neutral) hydrogen atoms with its electron in the n=2 (first excited) state. (Hint: at the effective temperature of the Sun, pretty much all of the

[1] Try saying that five times fast!

hydrogen is either in the ground state or first excited state and recall that the degeneracy of levels in the hydrogen atom is given by $g_n = 2n^2$)

(c) Explain the assumptions that went into getting this fraction!

3. Now let's talk about calcium. In the atmosphere of the Sun basically all of the calcium is CaII (i.e. singly-ionized). The ionization energy of calcium is 6.11 eV. Compare this energy to the hydrogen ionization energy and you'll see why the hydrogen is almost all neutral (HI) while the calciums is almost all singly-ionized (CaII). The CaII K line is the transition from the ground state to first excited state. The degeneracies of these levels are $g_1 = 2$ and $g_2 = 4$.

   (a) The wavelength of the CaII K line is 3933 Å. What is the ΔE of this transition in eV?

   (b) To determine the strength of the line, we need to find the fraction of all CaII atoms which have an electron in the ground state, so go for it. Again, make some simple assumptions to simplify your answer.

4. Now let's assume that the "strength" of a spectral line only depends on the fraction of atoms with electrons in the appropriate energy level, as well as the abundance of the atom creating the line. This isn't quite true, but the details are unimportant for us right now. Given the solar spectrum in Figure 1, assume that the calcium line is 400 times stronger than the hydrogen line. Using the fractions you calculated above, what are the relative abundances of the two elements?

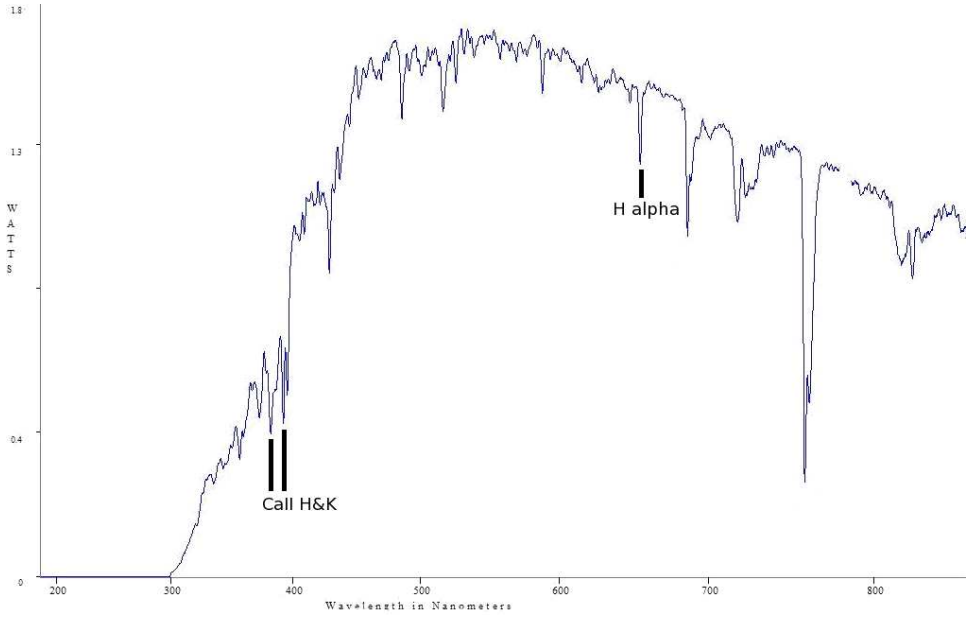

Figure 1: The region of the solar spectrum showing the CaII H & K lines and the Hα line.

5. In general, taking a spectrum of the entire blackbody curve of a star (i.e. the spectrum at *all* wavelengths) is very impractical. Usually, stellar spectroscopy involves a very small region of the spectrum, from which you can't usually determine the peak of the blackbody curve to find the star's temperature. Given a relatively small region of the spectrum, what is one way to determine both the temperature and abundance of an element simultaneously?