Ay 7A – Fall 2009
Section Worksheet 10
Come Sail Away, Come Sail Away, Come (Solar) Sail Away With Me

A solar sail is basically a large mirror attached to a spacecraft that reflects sunlight and uses the momentum of the incident photons to gently push the spacecraft along via radiation pressure. This can get a spacecraft up to greater speeds than more conventional chemical rockets and spacecraft equipped with a solar sail only need to carry a tiny amount of fuel (if any).

Solar sails should have a large area, since they should collect as much sunlight as possible (we'll show this in a minute), and should be light weight, since more massive sails will result in smaller accelerations (again, we 'll see this in a minute). They should also be durable since they must withstand the extremes of deep space travel. Most solar sails built so far have been made of thin, metal-coated, durable plastics such as Mylar or Kapton.

No solar sail has successfully flown in space as the primary propulsion system of a spacecraft, but government space agencies and private companies around the world have been testing this technology for quite awhile now. A recently build solar sail[1] had a thickness of a few microns (ordinary Saran Wrap is about 25 microns thick), an area of about 600 m^2, and a mass of 100 kg.

Let's think about how the solar sail actually works and see what kind of speeds we can get out of it.

1. The energy of a photon can be written as pc where p is the photon's momentum and c is the speed of light. It is this momentum that is transferred to the solar sail. Pressure can be written $P = p/A/t$ where A is area and t is time. Write an expression for the pressure of a photon in terms of the energy of the photon.

2. Now recall that flux can be written as the energy received per area A per time t. Rewrite the equation from the previous part in terms of flux.

3. We also know that we can write the flux from the Sun (which is the source of the photons that push our solar sail) in terms of its luminosity, $L_\odot$. Rewrite the equation from the previous part in terms of $L_\odot$ and distance from the Sun, r.

4. Now let's turn our expression for the radiation pressure from above into an expression for the radiation force on the solar sail, F. Call the area of the solar sail A.

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[1] Which failed to reach orbit when its booster didn't properly ignite.

5. Newton told us that $F = ma$. Using this, let's get an expression for the acceleration of the solar sail as a function of the variables given and fundamental constants. Notice how acceleration depends on the area of the sail and the mass of the sail.

6. If we launch a solar sail of area 600 m^2 and mass 100 kg into Earth orbit, what would its initial acceleration be? Recall that $c = 3 \times 10^{10}$ cm/s, $L_\odot = 4 \times 10^{33}$ erg/s, and 1 AU$= 1.5 \times 10^{13}$ cm.

7. If we send the spacecraft to Pluto, what would the instantaneous acceleration be at Pluto's orbit? Pluto has a semi-major axis of about 40 AU. HINT: Ratios are good things.

8. We know that the flux received from the Sun decreases as r^{-2} and thus the acceleration of the spacecraft will decrease as the spacecraft travels away from the Sun. Let's say the typical acceleration of the spacecraft on its way to Pluto is approximately the average of the two accelerations we calculated above. Using this approximation, how long would it take the solar sail spacecraft to get to Pluto from Earth? Assume it travels in a straight line, encounters no other planets, and Pluto is at opposition.

9. Finally, what is the average speed of the spacecraft as it travels from Earth to Pluto?